ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

31. Proposed by I. L. BEVERAGE, Monterey, Virginia.

"A man wishes to know how many hogs at $9, sheep at $2, lambs at $1, and calves at $9 per head, can be bought for $400, having of the four kinds, 100 animals in all. How many different answers can be given?"

[Satisfactory arithmetical solution desired.]

Solution by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Kidder Institute, Kidder, Missouri.

The average price per animal=$400÷100=$4.00.

		Dif.	Balance	Columns									
$4	$1	$3	5 lambs	5 sheep	3	10	17	24	31	38	45	52	59
	$2	$2	3 hogs & calves	2 hogs & calves	68	60	52	44	36	28	20	12	4
	$9	$5			29	30	31	32	33	34	35	36	37
			8	7									

For convenience of expression suppose the animals were bought at the prices named and sold at the average price $4. Then a lamb bought for $1 and sold for $4 is a gain of $3; a sheep bought for $2 and sold $4 is a gain of $2; a hog bought for $9 and sold for $4 is a loss of $5; and a calf bought for $9 and sold for $4 is a loss of $5. The gains on the sheep and lambs must be balanced by the losses on the hogs and calves. The L. C. M. of $3 and $5 is $15. If we gain $3 on one lamb to gain $15 we must take as many lambs as $3 is contained in $15, which are 5 lambs. If we lose $5 on a hog and a calf, to lose $15 we must take as many hogs and calves as $5 is contained in $15, which are 3 hogs and calves. In like manner, we find that we must take 5 sheep and 1 hog and 1 calf. Adding the balance columns, considering them as abstract numbers, we have 8 and 7. $8+7=15$. $100\div15=6\frac{2}{3}$. Multiplying the balance columns by $6\frac{2}{3}$ and adding the results horizontally, we get $33\frac{1}{3}$ lambs, $33\frac{1}{3}$ sheep and $33\frac{1}{3}$ hogs and calves, a result incompatible with the nature of the problem. Now it is clear that we must take a certain number of 8's + a certain number of 7's to make 100. Arithmetically, this can be done by trial only. We find by trial that two 8's and twelve 7's make 100. Hence, multiplying the column of 8 by 2 and the column of 7 by 12 and add the results horizontally we get 10 lambs, 60 sheep, and 30 hogs and calves. In like manner, we find that nine 8's and four 7's make 100. Multiplying the column of 8 by nine and the column of 7 by four, and adding the results horizontally we get 45 lambs, 20 sheep, and 35 hogs and calves. These are the only results that can be obtained by multiplying the balance columns by integral numbers. Since the numbers in the balance columns, when added horizontally give 5 as results, we may find a number of *fifths* times 8 and a number of fifths times 7

to make 100. By trial we find that $\frac{3}{5}\times 8+\frac{68}{5}\times 7=100$. Hence, multiplying the column of 8 by $\frac{3}{5}$ and the column of 7 by $\frac{68}{5}$ and adding the results horizontally, we get 3 lambs, 68 sheep, and 29 hogs and calves. In like manner, by multiplying the column of 8 by $\frac{10}{5}$ and the column of 7 by $\frac{60}{5}$, we get 10 lambs, 60 sheep, and 30 hogs and calves. By multiplying by $\frac{17}{5}$ and $\frac{52}{5}$ respectively, we get 17 lambs, 52 sheep, and 31 hogs and calves. The remaining six answers may now be easily written down without further trial. Since, in the first result there are 29 hogs and calves, we may have 1 hog and 28 calves, and so on. In all we may have 28 different results. In like manner in the second result, we may have 29 different results. In like manner 30 in third, 31 in fourth, 32 in the fifth, 33 in the sixth, 34 in the seventh, 35 in the eighth and 36 in the ninth. Hence, in all, we have $28+29+30+31+32+33+34+35+36=288$ different results.

For a fuller treatment of this class of problems see my *Mathematical Solution Book*.

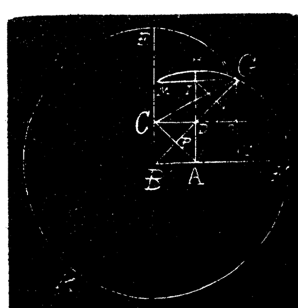
Correct solutions to this problem were received from *H. J. Geartner*, *Martin Spinx*, *John McDowell*, *P. S. Berg*, *P. C. Cullen*, and *M. A. Gruber*. *Mr. Gruber also gave a solution by Algebra.*

32. Proposed by P. C. CULLEN, Meade, Nebraska.

A horse is tied to a corner of a building 40 feet square, by a rope 110 feet long. Over how much land can he graze?

Solution by MARTIN SPINX, Wilmington, Ohio; Professor A. J. LILLY, Algona, Iowa; and Professor H. J. GEARTNER, Wilmington, Ohio.

Let $ABCD$ represent the barn, side $AB=40$ feet; BE represent the rope, length 110 feet; and $CG=70$ feet. Then the area of $FBEKF=\frac{3}{4}\pi\times 110^2$ $=28510.02$ sq. feet. The area of the two equal quadrants $AFGH$ and $CLGE$ $=\frac{1}{2}\pi 70^2=7696.84$ sq. feet. Now $AC=40\sqrt{2}$ feet; $PG=\sqrt{(GC^2-PC^2)}$ $=\sqrt{(70^2-800)}=10\sqrt{41}$ feet; and $DG=10\sqrt{41}$ $-20\sqrt{2}=10[\sqrt{41}-2\sqrt{2}]$ feet. $\therefore$ Area of square $ODIG=GD^2\div 2=\{10[\sqrt{41}-2\sqrt{2}]\}^2$ $\div 2=638.922932$ sq. ft. The height of the segment GNL is $LO=LD-OD=30$ feet$-\frac{1}{2}\sqrt{2}$ $GD=[30-(5\sqrt{82}-20)]$ feet$=5[10-\sqrt{82}]$ feet and the base is $GN=2GO=\sqrt{2}GD$ $10[\sqrt{82}-4]$ feet. Hence, according to the rule, p. 389, Ray's Higher Arithmetic, the area of the segments GNL and GMH is $2(LO^3\div 2GN+\frac{2}{3}LO\times GN)$ $=2\{[5(10-\sqrt{82})]^3\div 20[\sqrt{82}-4]+\frac{2}{3}\times 5(10-\sqrt{82})\times[10\sqrt{82}-4]\}=320.443888$ sq. ft. and the area of the half segments$=160.221944$ sq. ft. Hence, the area over which the horse grazes$=\frac{3}{4}FBEKF+FAH+LCE-(LOG+ODIG+GIH)=\frac{3}{4}\pi 110^2$ $+\frac{1}{2}\pi 70^2-(160.221944+638.922932)=35407.72514$ sq. ft.



This problem was solved in different ways and with different results by *P. S. BERG*, *John McDowell*, *G. B. M. Zerr*, and *D. G. Durrance*.

[For a demonstration of the rule referred to above, see my *Mathematical Solution Book*, pp. 201 and 202.—Ed.]